

WORDS TO KNOW

irrational number

Lesson 2

COMPARE AND ESTIMATE
IRRATIONAL NUMBERS NY-8.NS.2

INTRODUCTION

Real-World Connection

Luciano's company is building a wildlife sanctuary on a plot of land. The plot is square-shaped and has an area of 3 square miles. He must estimate the side lengths of the plot so he can purchase fencing. He will receive a discount on fencing if he buys more than 8 miles of fencing. Approximately how long is each side of the plot, in miles? Will he receive the discount on fencing? Let's practice the skills in the **Guided Instruction** and **Independent Practice** and see how Luciano solves his problem at the end of the lesson!



What I Am Going to Learn

- How to use rounding to estimate the value of irrational numbers
- How to use rational approximations of irrational numbers to compare the size of irrational numbers
- How to locate irrational numbers on a number line

What I May Already Know

- I know that numbers that are not rational are called *irrational*.
- I know how to evaluate square roots of small perfect squares.
- I know how to find, position, and order rational numbers on a number line.

Vocabulary in Action

- An **irrational number** cannot be written in the form $\frac{a}{b}$, where a and b are integers and b does not equal zero.

EXAMPLE

Look at the irrational number π . Which is greater, π or 3?

Step One Estimate the value of the irrational number.

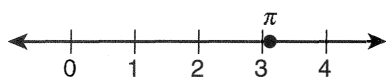
π is an irrational number because you cannot write it as a fraction.

You know that π is approximately 3.14.

Step Two Estimate the value of the expression, if needed, and then compare.

π is slightly greater than 3.

Here is the approximate location of π on a number line:

**THINK ABOUT IT**

The irrational number π is rounded to 3.14 but if not rounded, it continues on with no pattern in its decimals: 3.14159265359...
Is the number 3.14 irrational?
How do you know?

EXAMPLE

Which is greater, π^2 or 9?

Step One Estimate the value of the irrational number.

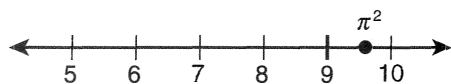
You know that π is approximately 3.14, which is greater than 3.

Step Two Estimate the value of the expression and then compare.

You also know that $3^2 = 9$.

Since π is slightly greater than 3, and $3^2 = 9$, then π^2 is greater than 9.

The number line shows the approximate location of π^2 using 3.1 as the estimate for π .

**TURN AND TALK**

On a number line, are irrational numbers always located between two whole numbers, or are they ever a whole number?

GUIDED INSTRUCTION

1. Estimate the value of $\sqrt{17}$ and plot its approximate location on a number line.

Step One Find perfect squares that are close to 17.

Since $4^2 = 16$, then $\sqrt{16} = 4$.

Since $5^2 = 25$, then $\sqrt{25} = 5$.

TIPS AND HINTS

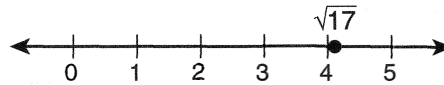
It may be helpful to memorize some of the perfect squares like 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

$$\pi = 3.14159265$$

Step Two Use the perfect squares to estimate the number.

Since 17 is slightly greater than 16, $\sqrt{17}$ is slightly greater than $\sqrt{16}$. Therefore, $\sqrt{17}$ is slightly greater than 4.

Step Three Show the number's position on a number line.

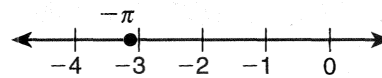


2. Which is greater, -3π or -9 ?

Step One Estimate $-\pi$.

π is about 3.14, or slightly greater than 3.

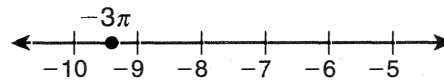
$-\pi$ is about -3.14 , or slightly less than -3 .



Step Two Estimate -3π .

Since $-\pi$ is to the left of -3 on a number line, then -3π will be to the

of -9 on a number line.



So, -9 is than -3π .

TIPS AND HINTS

Start by eliminating any number that you know is greater than 4.

3. Which number is less than 4?

(A) $\sqrt{18}$

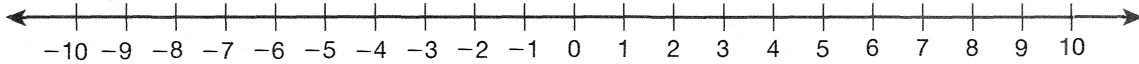
(B) $\frac{1}{2}\pi$

(C) $\sqrt{16}$

(D) 2π

Learning Together

Whether rational or irrational, every real number can be compared to rational numbers. Work with a partner to create a number line that is at least two feet long and ranges from -10 to 10 .



From a pile of rational and irrational numbers that also range from -10 to 10 , draw one card at a time and mark the number on the number line.

When you are finished, trade your number line with another team and verify that each other's number placements are correct.



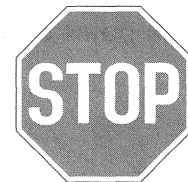
How Am I Doing?

What questions do you have?

Give three examples of irrational numbers that are greater than 5 but less than 7.

Sketch a number line and describe how to use it to compare negative irrational and rational numbers. For example, how would you use a number line to compare -3π and -12 ?

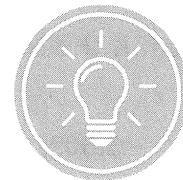
Circle the sign that shows how you are doing with the skill.



I am stuck.



I almost have it.



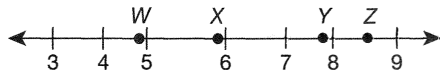
I understand the skill.

INDEPENDENT PRACTICE 1

1 Which irrational number has the greatest value?

- A π
- B $\sqrt{3}$
- C $\sqrt{11}$
- D 3.151151115...

2 Look at the number line below.



Which point on the number line below **best** represents $\sqrt{33}$?

- A W
- B X
- C Y
- D Z

3 Sarki wrote the numbers $\frac{\pi}{2}$, 1.7, $-\sqrt{27}$, and -3.509 in order from least to greatest. Which is the correct order?

- A $-3.509, -\sqrt{27}, \frac{\pi}{2}, 1.7$
- B $-\sqrt{27}, -3.509, 1.7, \frac{\pi}{2}$
- C $-\sqrt{27}, -3.509, \frac{\pi}{2}, 1.7$
- D $-\sqrt{27}, \frac{\pi}{2}, 1.7, -3.509$

TIPS AND HINTS

Estimate the value of each expression as a decimal number so that they are easy to compare.

TIPS AND HINTS

A "point on a number line" is always relative to the visible numbers on either side of it.

SKETCH IT

It might help to put the numbers on a number line. When sketching a number line, use a ruler to make equally spaced intervals.

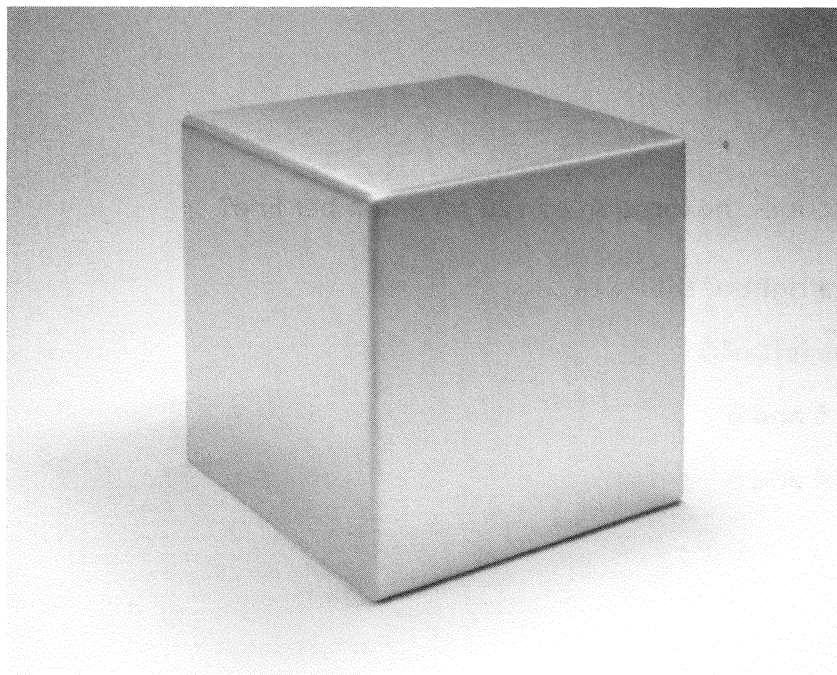
4

A cube can hold exactly 256 cubic centimeters. About how long is one of its edges, rounded to the nearest tenth of a centimeter?

Explain your answer.

TIPS AND HINTS

Be sure to explain both your answer and the reasoning you used.



INDEPENDENT PRACTICE 2

1 Which expression has a value greater than 8?

- A 2π
- B $\sqrt{59}$
- C $\sqrt{65}$
- D $\pi + 4$

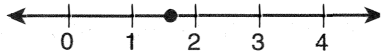
2 Which number is greater than 10?

- A $\sqrt{13}$
- B 0.5π
- C 4π
- D $\sqrt{24}$

3 Which **best** describes the location of $\sqrt{26}$ on a number line?

- A just to the right of 6
- B just to the left of 5
- C between 5 and 6
- D between 4 and 5

- 4 An approximate location for the value of an expression is plotted on the number line below.



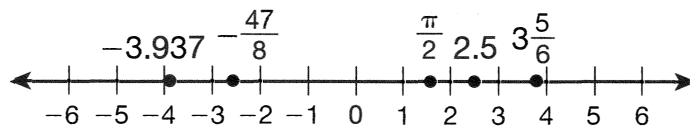
The point on the number line could represent which expression?

- A 2π
- B $\frac{1}{2}\pi$
- C $2\sqrt{5}$
- D $\sqrt{8}$

- 5 Which statement about $-\sqrt{30.25}$ is true?

- A $-\sqrt{30.25}$ is greater than 5, but less than 6.
- B $-\sqrt{30.25}$ is less than -6 .
- C $-\sqrt{30.25} = -5.5$.
- D $-\sqrt{30.25}$ cannot be ordered because it is not a real number.

- 6 Ashanti placed the numbers $\frac{\pi}{2}$, 2.5 , $-\frac{47}{8}$, -3.937 , and $3\frac{5}{6}$ on the number line from least to greatest.



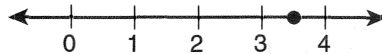
Did she do it correctly?

- A Yes, they are in the correct order.
- B No, $\frac{\pi}{2}$ is greater than 2.5 .
- C No, $-\frac{47}{8}$ is less than -3.937 .
- D No, $3\frac{5}{6}$ is less than $\frac{\pi}{2}$.

7 Tori needs to order the numbers -7.93 , -8.241 , $\frac{17}{9}$, 9.76 , $-\frac{76}{9}$, and π . What would be the **best** strategies for her to use?

- A She should convert each number to an equivalent fraction, and then place all of them on a number line.
- B She should convert all the numbers to fractions with like numerators, and then put them in order from greatest to least.
- C She should convert each number to a decimal and compare the largest place values.
- D She should convert everything to a decimal and compare the smallest place values.

8 Jorge plotted the approximate location of $\sqrt{8}$ as shown on the number line below.



Why is the approximate location Jorge plotted incorrect?

Explain your answer.

Between which two whole numbers should $\sqrt{8}$ be placed?

Answer between _____ and _____

9 Name three irrational numbers that are between 11 and 12. Approximate the value of each one to the nearest tenth.

Explain your answer.

EXIT TICKET

NY.8.NS.2

Now that you have mastered comparing and estimating irrational numbers, let's solve the problem in the Real-World Connection.

Luciano's company is building a wildlife sanctuary on a plot of land. The plot is square-shaped and has an area of 3 square miles. He must estimate the side lengths of the plot so he can purchase fencing. He will receive a discount on fencing if he buys more than 8 miles of fencing.

To find the length of each side, he must estimate the value of $\sqrt{3}$. What is an estimate for the value of $\sqrt{3}$? Approximately how long is each side?

What is the total amount of fencing Luciano needs? Will he receive the discount on fencing?

